

The Tensor Track: an Update

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The *tensor track* approach to quantum gravity,¹ is based on a new class of quantum field theories, hereafter called tensor group field theories (TGFTs).^{2–6} We provide a brief review of recent progress and list some desirable properties of TGFTs. In order to narrow the search for interesting models, we also propose a set of guidelines for TGFT's loosely inspired by the Osterwalder-Schrader axioms of ordinary Euclidean QFT.

Keywords: tensor models, quantum gravity, group field theory, axiomatic field theory

1. Introduction

String theory and loop quantum gravity (LQG), the two leading approaches to quantum gravity, are currently stuck by a common problem: the lack of a convincing second-quantized non-perturbative formulation.

About twenty years ago, d -branes with $d \geq 3$ were recognized as key features of string theory. The non-perturbative framework that should explain the presence of branes and their beautiful associated dualities was called M -theory, where M means *matrix*, *mystery* or *magic*. But a simple action for this M -theory is still missing. This problem may be related to the huge and puzzling *landscape* of perturbative ordinary string vacua. The way forward may require some radical simplification.

A candidate for a non-perturbative second quantized formulation of LQG was quite early identified⁷ as group field theory (GFT).^{8,9} But GFT developed slowly and is not yet the mainstream formulation of LQG; in particular its correct combinatorics and renormalization have been found only recently.

The *tensor track* is a generalization of the random matrix approach to

the quantization of two-dimensional gravity. It rebuilds early tensor models¹⁰ and GFT around new principles derived from the universal properties of general random tensors.¹¹ It leads to a new class of quantum field theories which successfully renormalize GFT divergences, now correctly interpreted as ultraviolet rather than infrared in the Wilsonian sense*.

Properly supplemented with standard-model matter fields (and possibly supersymmetry?), this approach may hopefully some day relate different approaches such as LQG and superstrings through a framework that we could nickname *T*-theory (*T* like *tensor* or *total*). Indeed tensor models contain many embedded matrix models (their *jackets*¹³). They have therefore at least in principle the potential to quantize strings and higher dimensional branes on the same footing, leading to simpler models.

2. Basic Hypotheses

In the absence of direct experimental evidence, we expect the search for a good theory of quantum gravity to remain speculative and based on analogies for quite a while. The tensor track emphasizes quantum field theory, Feynman functional integrals, phase transitions and the Wilsonian renormalization group. Hence it reflects certain prejudices. Other approaches emphasize other concepts, such as the unification of all interactions, extended symmetries, canonical quantization, lattice regularizations etc...

Nevertheless the tensor track is rooted in deep convictions. Quantum field theory, functional integrals, phase transitions and the Wilsonian renormalization group together form our most advanced and most successful tools to understand physical systems with *many degrees of freedom*[†]. Only quantum field theory together with renormalization can compute accurately (more than ten digits!) physical effects which involve radiative corrections. Only quantum field theory has successfully renormalized all other interactions. Although gravity around a flat Minkowski background is not renormalizable in the perturbative sense, still the most conservative option seems to enlarge quantum field theory in a suitable minimal way to quantize it.

The tensor track bets upon the idea that quantum gravity should be background independent, even topology-independent, and that classical

*See¹² for renormalization of spin-foams based on lattice-like coarse-graining.

[†]We should in particular certainly not consider quantum field theory as just a way to combine special relativity and quantum mechanics, nor renormalization as a way to hide infinities. They are far more universal, as exemplified by their great success, first of course in particle physics but also in condensed matter, which is not relativistic, in statistical mechanics, which is not quantum, etc.

space-time and general relativity are effective concepts emerging from a more fundamental theory through one or several *phase transitions*, nicknamed geometrogenesis.¹⁴

Indeed phase transitions, whose modern understanding is provided by the Wilsonian renormalization group, are generic features of physical systems with many degrees of freedom. In fact the myriad of phase transitions leading to composite structures is perhaps the most obvious characteristic of our universe. Why would the geometry of the universe itself, with its huge number of degrees of freedom, not follow this trend? Another observation is that phase transitions occur at particular scale, hence they could provide an explanation for the existence of the Planck and cosmological constant scales in our universe. Finally of course geometrogenesis nicely fits with the big-bang, as they could be just the same thing.

The main *new* (hence perhaps most controversial) *bet of the tensor track* is to replace the ordinary principle of local interactions by an extended notion based on tensor invariance[†]. But again this choice is not arbitrary. We believe that quantizing gravity is essentially the same as correctly randomizing geometry. Since our universe is very large, we need a robust tool to perform a statistical analysis of *large geometries in three and four dimensions*. The most fundamental tool in probability theory is the law of large numbers and the central limit theorem. The theory of random matrices and of their invariant interactions provides the equivalent of this tool to analyze two dimensional geometries. The recently discovered theory of higher rank random tensors and of their invariant interactions¹¹ is the natural candidate for their generalization to dimensions three and four.

3. GFT's and TGFT's

Consider a (simple, compact) Lie group G , endowed with its natural Haar measure and metric. Complex valued square integrable functions on G form an associated Hilbert space $H(G)$. The group structure on G allows Fourier analysis. $H(G)$ is infinite-dimensional, and admits various approximation

[†]One of the few consensual ideas on the subject is that ordinary locality should be extended to quantize gravity. Spatial distances or areas smaller than the Planck scale cannot be measured in the usual way, as the measuring probes would disappear into the black hole created by their own gravitational field. However this does not mean that meaningful physics necessarily stops at that scale. Transplanckian *scales* could exist in the renormalization group sense even when there is no longer any well-defined notion of *distance*;¹ and transplanckian *physics* might be detected through indirect but convincing effects, which may lead to future predictions and physical discoveries.

schemes through finite N -dimensional vector spaces $H_N(G)$. Rank d tensor fields are defined as elements of the external tensor product of d copies of $H(G)$, or of $H_N(G)$, in which case we are interested in letting the cutoff N tend to infinity[§]. The scalar product in $H(G)$ or $H_N(G)$ allows to raise and lower tensor indices, hence to contract indices at identical positions between a tensor and its complex conjugate.

Equivalently a tensor field can be considered as a function on the product G^d of d copies of G , but this erases its tensor aspect. Group field theory (GFT) nevertheless emphasizes this second point of view; it is defined by an action for fields living on G^d .⁹ In the initial example of group field theory, the Boulatov model,⁸ the group G is $SO(d)$ or its universal covering group and the field incorporates a projection which averages over a common group translation of the d variables. This projection trivializes the holonomies along the faces of any Feynman amplitude, hence it implements the BF action on the 2-complex corresponding to the Feynman graph.⁸ However the usual vertex envisioned by ordinary GFT is not a tensor invariant (in the precise sense defined below) and does not correspond to a stable action. The theory triangulates very singular pseudo-manifolds in addition to regular manifolds. No $1/N$ expansion has been found to organize the amplitudes of this theory, and although many amplitudes become infinite in the no-cutoff limit, they could not be properly *renormalized*.

This situation changed with the discovery of *colored* group field theories¹⁵ and of their associated simpler random tensor models.¹³ They triangulate better behaved spaces¹⁶ and admit a $1/N$ expansion¹⁷ (where N is the ultraviolet cutoff). Their uncolored formulation^{11,18} rests on the classification of all the $U(N)^{\otimes d}$ -invariant interactions of a pair of complex conjugate random tensors. It generalizes the standard invariance of (Wishart) matrix models. A welcome property is that such interactions are also often *stable* for a suitable sign of the coupling constant, curing one of the main problems of GFT.

The tensor track proposal conjectures that *this* tensor invariance should be the proper extension of the ordinary notion of *locality* needed to quantize gravity. Rebuilding GFT to incorporate this tensorial aspect of the field, we obtain a new class of quantum field theories, namely the TGFTs.^{2,6}

However just as locality in quantum field theory is fundamental but is only an exact property of interactions, not of propagators, we expect in-

[§]As usual in QFT, tensor fields may be in fact *distributions* rather than functions but we skip this technicality here.

interesting TGFT's to have non- $U(N)^{\otimes d}$ -invariant propagators. It is in fact the interplay between approximately local propagators and local interactions which launches the renormalization group flow of coupling constants in quantum field theory, and the same happens in TGFTs.

4. Desirable Properties

4.1. Renormalization

Just renormalizability is a property shared by all physical interactions except (until now!) gravity. In the renormalization group sense it is *natural*. Indeed just renormalizable interactions survive long-lived RG flows. They can be considered the result of a kind of *Darwinian selection* associated to such flows. Therefore if quantum gravity can be renormalized as proposed in^{1,19} it will rely on the same powerful technique that applies successfully to all other interactions of the standard model. There will be no longer any need for a teleological or anthropic interpretation.

The simplest renormalizable TGFT has been found in dimensions three and four for the $U(1)$ group.^{2,4} In dimension 4 it has two unexpected ϕ^6 -like marginal interactions, hence a richer RG flow than the usual ϕ^4 models.³

4.2. Asymptotic Freedom

Again asymptotic freedom is a property shared by all physical interactions except (until now!) gravity. Indeed QCD is asymptotically free and the electromagnetic sector inherits at high energy the asymptotic freedom of the unified electroweak theory[¶].

Asymptotic freedom is desirable to build a geometrogenesis scenario for TGFTs,^{14,20–22} and in fact may be generic in the world of tensors of rank greater or equal to three. It has been *already established* for the simplest renormalizable TGFTs in dimension 3 and 4.^{3,4} The new locality axiom allows wave function renormalization to compete with coupling constant radiative corrections, and typically to win in the case of rank ≥ 3 tensors. Recall that absence of asymptotic freedom is the rule for the simplest models of scalar, vector and matrix type (except of course non-Abelian gauge theories) and that asymptotic safety is barely reached for natural matrix field theories such as the Grosse-Wulkenhaar model.²³ In addition, the infrared growth of the coupling constant occurs for the *stable* sign of the

[¶]The ultraviolet behavior of the Higgs sector is a subtle issue not considered here.

interaction, hence may lead to the discovery of singularities which could represent *unitary matter*. This would improve on single-matrix model singularities which lead to (non-unitary) Lee-Yang type singularities.

4.3. Constructibility

Constructibility of a quantum field theory means that its perturbative series can be uniquely resummed (typically through a kind of Borel resummation).^{26,27} Physically it is related to *stability and uniqueness of the vacuum*. It guarantees that at least the perturbative phase of the theory is unique and mathematically well-defined at small coupling.

TGFTs with stable positive interactions should be constructible, and the corresponding proofs seem doable, thanks to a new constructive tool called the loop vertex expansion (LVE),²⁸ adapted to the extended notions of locality that govern matrix or tensor models. Significant results have been already obtained in this direction.^{11,29} We expect the full Borel summability of renormalizable asymptotically free TGFTs to be more difficult than those of infrared ϕ_4^4 and of the Gross-Neveu model,²⁷ but much simpler than the corresponding study for non-Abelian gauge theories.

The existence of such a constructive perspective is a very important *long term asset* of the tensor track program, which (to our knowledge) is missing in all other current approaches to quantum gravity.

4.4. Geometricity

We are ultimately interested in models whose effective infrared physics leads naturally (under suitable boundary conditions) to our universe, namely a large quasi-flat four dimensional space-time with a metric obeying the (classical) Einstein equations.

Recently models were developed which incorporate the constraint projector of the BF theory. This could lead to a geometrogenesis with a smoother metric. The first four-dimensional models of this type have been proved superrenormalizable on the $U(1)$ group.⁶ We expect ϕ^6 models in dimension 3 and on the $SU(2)$ group to be just renormalizable.²⁵

To guide geometrogenesis towards the desired outcome in four dimensions we may have to decorate the most natural renormalizable TGFT's with additional *geometric conditions*. Spin foam models, in particular the 4d models which incorporate Plebanski simplicity constraints²⁴ could inspire such decorations. We are open to other possibilities, as the only rule is to find the *simplest* such models with gravity as their effective limit.

4.5. *Dualities, holography*

Dualities such as Born duality, Langmann-Szabo dualities in the Grosse-Wulkenhaar matrix models or the many dualities of string theory may have interesting analogs in the TGFT world. Such dualities could allow integrability and exact solvability of particular TGFT models. This possibility should be systematically investigated.

Similarly it might be interesting to incorporate some kind of holographic principle in TGFT's. The structure of the boundaries of TGFT amplitudes, which are themselves lower rank TGFT vacuum amplitudes, suggests some principle of this kind.

5. Rules for TGFT's

We sketch now tentative rules for TGFTs that could later evolve into a true axiomatic scheme. Axioms embody the long term reflection of the scientific community on the most fundamental aspects of quantum field theory and are therefore a valuable source of inspiration into unexplored territory such as quantum gravity. But at this early stage we intentionally formulated our proposal in a non-technical language. It should not be considered rigid nor exclude interesting future developments (for instance Fermionic axioms etc...). The hard work, which remains entirely to be done, requires a more precise mathematical formulation of these rules and the proof that interesting interacting TGFTs indeed obey them.

According to our conservative analogy-based approach we search for natural analogs of the main axioms of Euclidean quantum field theory. These new rules should imply a new kind of constructive program, for TGFTs. The initial constructive program^{26,27} is far from complete, as it does not include yet the full construction of the four dimensional Yang-Mills theories. However a constructive program for TGFTs could actually progress faster in the coming years, since interesting asymptotically free models may be free of subtle constructive issues such as Gribov ambiguities which plague the ordinary non-Abelian gauge theories.

Our rules are formulated in terms of approximation schemes based on limits of functional integrals with cutoffs. In ordinary quantum field theory we know that each cutoff violates some axiom; but uniqueness of the limit typically ensures that the theory without cutoffs satisfies all of them.

Rule 1: Tensor Invariance and Positivity of Interactions

This rule replaces locality. The fields are considered both as functions on G^d , the *pre-space*, and as rank- d tensors on $H(G)$. The bare functional

measure $d\nu(\phi, \bar{\phi})$ of a TGFT should be formally of the form

$$d\nu(\phi, \bar{\phi}) = \frac{1}{Z} d\mu_C(\phi, \bar{\phi}) e^{-S_{int}(\phi, \bar{\phi})}, \quad S_{int}(\phi, \bar{\phi}) = \sum_{b \in \mathcal{B}} \lambda_b I_b(\phi, \bar{\phi}), \quad (1)$$

where C is the covariance or bare propagator, \mathcal{B} is a finite set of *connected positive tensor invariants* labeled by b , and the coupling constants $\lambda_b \in \mathbb{C}$, should have positive real parts. The non formal definition requires as usual to introduce an ultraviolet cutoff N , then to control the limit $N \rightarrow \infty$.

Tensor invariants are obtained by convolution of a set of fields φ and $\bar{\varphi}$, in such a way that the k -th index of a field φ is always contracted with the k -th index of a conjugate field $\bar{\varphi}$, resulting in a polynomial invariant under $U(N)^{\otimes d}$. They are canonically represented by closed bipartite d -colored graphs: each field φ (resp. $\bar{\varphi}$) is represented by a white (resp. black dot), and each contraction of a k -th index between two fields is pictured as a line with *color* label k linking the two relevant dots (see Figure 1). *Connected* invariants correspond to connected graphs. *Positive* invariants admit a mirror symmetry allowing to write them as sums of moduli squares. For instance in Figure 1 the first two tensor invariants admit such a symmetry, but not the last one. The conditions on positive real parts for the couplings λ_b ensure stability of the corresponding action.

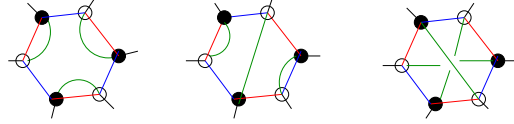


Fig. 1. Some connected tensor invariants in $d = 3$

Rule 2: Discrete Permutational Symmetry

We suggest to replace continuous rotational and translational invariance by a discrete permutation invariance, as appropriate in case of geometric discretizations. Hence we require that the Schwinger functions should be invariant under the discrete symmetry group Σ_d with $d!$ elements. In particular tensorial interactions should be symmetrized over the discrete permutations of the d space-time colors. This implies constraints on the coupling constants: they should be equal for invariants which differ only by a permutation of colors.

Rule 3: Clustering

The clustering axiom in ordinary QFT requires the Schwinger functions to decay as their external arguments are taken apart. In the pregeometric tensor world external arguments of rank d models represent boundaries which are themselves colored tensor models of rank $d - 1$. There is not any good notion of distance yet, hence we should rather ask for a decay in the defining parameters of the sum $S_r(c, d_1, \dots, d_c)$ of Feynman amplitudes which have r external legs defining a boundary with c connected components, each having *degree* d_i .

In case of exponential decay, this would mean that there exist constants $K > 0$ and $\epsilon > 0$ such that

$$|S_r(c, d_1, \dots, d_c)| \leq K e^{-\epsilon(r+c+\sum_{i=1}^c d_i)} \quad (2)$$

The number of connected components of the boundary and the sum of their degrees gives some measure of the complexity both of the topology and of the cellular structure of that boundary. Decay of this type holds at the perturbative level for the models considered so far.² Beware however that the notion of *connectedness* may depend on the models considered.⁶

Rule 4: Positivity and Mirror-Positivity of Propagator

The only quadratic invariant for tensors is the mass term which is also local. Hence *locality coincides with tensor invariance for the 2-point function*. We want to consider a bare propagator which softly breaks both of these invariances, in order to launch a renormalization group flow. We require that it should have a non-trivial positive spectrum, allowing its parametric representation as $C = \int_0^\infty e^{-\alpha C^{-1}} d\alpha$. For renormalization group analysis to work we also need that it should become approximately local in the ultraviolet regime $\alpha \rightarrow 0$.

Most controversially perhaps, we suggest that an analog of Osterwalder-Schrader positivity or of the Markov property of ordinary Euclidean fields should also hold for TGFTs. Indeed this key property of Euclidean fields allows the continuation to Lorentzian time and the unitary time evolution of states. Having an analog of that in the tensor world could hopefully also lead to a time interpretation of the resulting effective theory. Without it we could build arbitrarily many convergent quantum field theories in four dimensions, namely non-unitary theories with *ultraviolet cutoffs*. We certainly want to exclude these.

Assuming convergence problems solved through constructive theory, any Euclidean quantum field theory with OS-positive bare propagator and *local interactions* is fully OS positive.²⁶ Since the interactions we consider are non-local, we have not found yet any analog of this result for tensors. But

we think interesting, at least as a heuristic and tentative rule, to suggest the bare propagator of our theory to be mirror-positive in the following sense. The pre-space G^d , thanks to the group structure of G , comes equipped with d fundamental involutions, or mirror symmetries

$$(g_1, \dots, g_i, \dots, g_d) \rightarrow S_i(g) = (g_1, \dots, g_i^{-1}, \dots, g_d).$$

We could then define a generalized OS-positivity of the propagator C , which mean that the p by p matrix with matrix elements $C(g_k, S_i(g_l))$ should be positive for any $i \in \{1, \dots, d\}$ and any finite collection of p arguments $\{g_k\}$ in G^d . This property holds for propagators admitting a Euclidean Källén-Lehmann representation

$$C = \int_0^\infty \frac{d\rho(m)}{-\Delta + m}, \quad (3)$$

where Δ is the Laplacian on G^d and $d\rho(m)$ a positive measure. This representation excludes better ultraviolet behavior than the one of the Laplacian.

Another strong argument for the Laplacian as propagator in the pre-space comes from the Taylor expansion around divergent two-point functions required by renormalization.³⁰ Ultimately the Laplacian is a natural choice on G , which as a (simple, compact) Lie group comes equipped with a differential structure and a metric, so TGFTs should use it.

6. Conclusion

Renormalizable 3D and 4D TGFTs *exist*. The most natural models are asymptotically free, an encouraging fact for geometrogenesis scenarios. Adding some pre-geometric content is possible at least in some cases (eg $U(1)$ in $D = 4$, probably $SU(2)$ in $D = 3$). Axiomatic schemes can be considered for TGFTs, leading to a new constructive program. Simplified models have been proved Borel-summable using the LVE.²⁸

The main open problem is to analyze geometrogenesis of natural renormalizable TGFTs and to find the right pre-geometric content that would lead to general relativity as effective lower energy physics in dimension 4.

Success, for probably a long time to come, will be measured in terms of how far these attempts can be pushed on the mathematical level, how convincingly they imply general relativity as effective limit and how many applications to different domains they bring. In this last respect the tensor track is promising, as it is linked to a growing list of applications of random tensors to statistical mechanics.³¹

Acknowledgments

This paper has greatly benefitted from many discussions with J. Ben Geloun, S. Carrozza, R. Gurau and D. Oriti. I would like to thank the organizers of the XXIX International Colloquium on Group-Theoretical Methods in Physics and the Perimeter Institute for support.

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